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Control and dynamics approach to robust quantum computing

ABSTRACT

During the entire performance period, from 12 May 2003 through 31 December 2006, we have conducted theoretical and computational research on quantum control problems central to quantum computation. In particular we completed a thorough and rigorous analysis of feedback-stabilization of entangled state preparation with atomic hyperfine spins, based on continuous Faraday rotation measurement and feedback via global magnetic fields. We connected three distinct layers of modeling, from Hamiltonian dynamics (quantum field theory) to quantum trajectory models to nonlinear stochastic control theory. We also initiated work on a control-theoretic description of quantum memories based on stabilizer coding and continuous syndrome measurement.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

"Modeling and feedback control design for quantum state preparation"
Ramon van Handel, John K. Stockton and Hideo Mabuchi
J. Opt. B: Quantum Semiclass. Opt. 7 S179-S197 (2005)
"Quantum projection filter for a highly nonlinear model in cavity QED"
Ramon van Handel and Hideo Mabuchi
J. Opt. B: Quantum Semiclass. Opt. 7 S226-S236 (2005)
"Feedback control of quantum state reduction"
Ramon van Handel, John K. Stockton and Hideo Mabuchi
IEEE Trans. Automat. Control 50, 768-780 (2005)
P. Chigansky and R. van Handel, "Model robustness of finite state nonlinear filtering over the infinite time horizon," Anal. Appl. Probab. 17, 688 (2007)
M. Mirrahimi and R. van Handel, "Stabilizing feedback controls for quantum systems," SIAM J. Control Optim., in press (2007)

Number of Papers published in peer-reviewed journals: 5.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

H. Mabuchi, "Quantum parameter estimation with atomic hyperfine spins," quantum information sciences seminar at MIT, 12/4/06

H. Mabuchi, "Quantum parameter estimation with cold atomic spins," invited talk at "Quantum Communication, Measurement and Control" in Tsukuba, 11/29/06

H. Mabuchi, "Control theory meets quantum optics/info: modeling tools for quantum nonlinear dynamics," Center for Advanced Studies seminar at University of New Mexico, 11/9/06

H. Mabuchi, "Applications of quantum filtering and feedback," University of Toronto, 9/14/06

H. Mabuchi, "Wonham filters and stabilizer codes," Principles and Applications of Control in Quantum Systems, Harvard, 8/8/06

H. Mabuchi: "Applications of quantum filtering and feedback"
Workshop on Quantum Probability, Information and Control
University of Nottingham, 17 July 2006

H. Mabuchi: "Applications of quantum filtering and feedback"
4th International Workshop on Optimal Control of Quantum Dynamics
Schloss Ringberg, 6 December 2005

R. van Handel and H. Mabuchi: "Control-theoretic analysis of quantum error tracking based on coding and continuous syndrome measurement"
Frontiers in Optics / OSA Annual Meeting, 18 October 2005

"Deterministic Preparation of Spin-Squeezed States via Real-Time Quantum Feedback," J. M. Geremia, DFG Focus Meeting, Bad Honef, Germany (1/04).

"Real-time quantum feedback control," Hideo Mabuchi, SAMSI Workshop on Multiscale Modeling and Control Design, Research Triangle Park (1/04).

"Real-Time Quantum Feedback Control in Cold Atoms," J. M. Geremia, UC Berkeley, Berkeley, CA (2/04).

"Real-Time Quantum Feedback Control: Deterministic State Reduction in Cold Atoms," J. M. Geremia, SQuInT 2004, UC San Diego, San Diego, CA (2/04).

"Feedback Control of Continuous Projective Measurement," (poster) J. Stockton, SQuInT 2004, UC San Diego, San Diego, CA (2/04).

"Knowing what you know: estimation and control in nanoscale systems," Hideo Mabuchi, MIT Center for Bits and Atoms Colloquium, Cambridge, MA (3/04).

"Quantum measurement and feedback control with cold atoms," Hideo Mabuchi, Harvard/MIT Center for Ultracold Atoms Seminar, Cambridge, MA (3/04).

"Real-Time Quantum Feedback Control in Cold Atoms," J. M. Geremia, University of Oregon, Eugene, OR (3/04).

"Collective Spin State Preparation with Quantum Measurement and Control," J. K. Stockton, Workshop on Control of Quantum Mechanical Systems, UC Berkeley, Berkeley, CA (4/04).

"Feedback Control of Quantum State Reduction," R. van Handel, 8th Southern California Nonlinear Control Workshop, UCSB, Santa Barbara, CA (5/04).

"Quantum measurement and feedback with atomic spins," Hideo Mabuchi, APS Division of Atomic, Molecular, Optical, and Plasma Physics, Tucson, AZ (5/04).

"Quantum Measurement, Entanglement and Feedback," J. M. Geremia, Physics Department Seminar, California State University Long Beach, Long Beach, CA (6/04).

"Feedback control of quantum state reduction," H. Mabuchi, FOCUS -- Building Computational Devices Using Coherent Control, University of Michigan, Ann Arbor, MI (6/04).

"Feedback Control of Continuous Projective Measurement," (poster) J. Stockton, FOCUS -- Building Computational Devices Using Coherent Control, University of Michigan, Ann Arbor, MI (6/04).

"Feedback control for quantum and classical uncertainty management," Hideo Mabuchi, Connections: 50th Birthday Celebration for John Doyle, Pasadena, CA (7/04).

"Feedback Control of Continuous Projective Measurement," (poster) J. Stockton, Fields Institute Meeting on Quantum Information and Quantum Control, University of Toronto, Toronto, Canada (7/04).

"Feedback stabilization of quantum entangled-state preparation," Hideo Mabuchi, Fields Institute Meeting on Quantum Information and Quantum Control, Toronto, CA (7/04).

"Real-time feedback control of quantum state reduction," Hideo Mabuchi, XIX International Conference on Atomic Physics (ICAP 2004), Rio de Janeiro, Brazil (7/04).

"Microcavities: strong coupling of atoms and photons," H. Mabuchi, Photonics Technologies Advancement Program Workshop on Optical Microcavities, San Diego (7/15/03).

"Coherence in broadband atomic magnetometry," H. Mabuchi, Tokyo University, Tokyo, Japan (9/15/03).

"Continuous observation of open quantum systems," H. Mabuchi, U.S.-Japan Joint Workshop on Coherencein Quantum Systems, Yatsugatake, Japan (9/17/03).

"Quantum filtering and broadband atomic magnetometry," H. Mabuchi, EURESCO Conference on Quantum Optics, Granada, Spain (9/28/03).

"Identification, modeling, and control of quantum and bio-molecular systems," H. Mabuchi, Workshop on New Horizons in Molecular Sciences and Systems: An Integrated Approach, Okinawa, Japan (10/17/03).

"Deterministic preparation of spin-squeezed states via real-time quantum feedback," H. Mabuchi, Stanford-ENS Quantum Entanglement Symposium, Stanford, CA (12/16/03).

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(d) Manuscripts

R. van Handel and H. Mabuchi, "Optimal error tracking via quantum coding and continuous syndrome measurement," quant-ph/0511221

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Graduate Students

NAME	PERCENT_SUPPORTED	
Ramon van Handel	1.00	No
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED	
Lucas Bouten	0.50	No
FTE Equivalent:	**0.50**	
Total Number:	**1**	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Ramon van Handel	No
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

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Final Report for DAAD19-03-1-0073
Control and Dynamics Approach to Quantum Computing

Hideo Mabuchi – Physical Measurement and Control – California Institute of Technology

STATEMENT OF THE PROBLEM STUDIED

During the performance period we thoroughly addressed one major problem, feedback-stabilization of conditional quantum state preparation, and began work on a second, quantum error correction via continuous syndrome measurement. Below we describe these two problems in turn.

Quantum state preparation: It has long been appreciated that conditioning after a (non-destructive) measurement can lead to the creation of interesting quantum states, including entangled states. The main appeal of this strategy for quantum state preparation in a quantum computing context is that it generally does not require controlled quantum gates among the qubits in order to generate entanglement of a known and structured form. Rather, each qubit interacts with a quantum ancilla of some kind (in practice usually an optical field) and after the ancilla is projected the qubits can be left in an entangled state. One can think of this process as implementing a kind of 'second order' interaction among the qubits.

One drawback of conditional state preparation is that there are multiple possible results for the ancilla measurement, each of which will leave the qubits in a different post-measurement state. In most concrete scenarios it tends to be the case that the more entangled outcomes are less probable.

Our aim in the ARO-sponsored research was to demonstrate theoretically (as well as via simulation) that real-time feedback could be used to make the outcome of a conditional state preparation procedure deterministic. We wanted to make use of the insight that real measurements are actually spread over some time interval, and that it is often realistic to apply global control Hamiltonians to the qubits (which can be much easier than applying pairwise quantum gates) during the mea-

surement process and conditioned on the partial measurement results. Our technical approach was to cast the conditional state preparation procedure, with feedback, into the formal language of control theory and to see what rigorous results we could obtain.

Quantum error correction: Our successes in the state-preparation problem led us to ask whether real-time feedback analysis could also be used for the stabilization of an unknown quantum state against decoherence. We wanted to build upon earlier work by Ahn, Doherty and Landahl [1], which considered a continuous-time 'relaxation' of stabilizer quantum coding and cast the problem into control-theoretic formalism. Our main goal for the preliminary stage of work was to explore whether a continuous-time model could shed light on how the familiar concepts of discrete-time quantum error correction could be adapted to a more realistic scenario of dynamic decoherence, finite-strength measurement and finite-strength recovery operations.

SUMMARY OF THE MOST IMPORTANT RESULTS

The bulk of our results were obtained on the problem of conditional state preparation; we will describe these first. The preliminary results we obtained on the quantum error correction problem were quite promising and led to the proposal and award (from ARO/DTO) of a new project on quantum memories based on finite-strength syndrome measurement. Below we will describe the initial points of progress that are credited to support from DAAD19-03-1-0073.

Quantum state preparation: The main accomplishment of our work has been the vertical integration of several distinct, crucial 'layers of modeling' in the problem of feedback-stabilized conditional quantum state preparation. At the most abstract level we have nonlinear stochastic control theory, which tells us how to prove the absolute stability (and sometimes the robustness) of feedback protocols, *given that* we have started from correct modeling equations. At the middle level of abstraction we have the quantum trajectory formalism of quantum optics, which has become a common theoretical tool for modeling experimental scenarios involving continuous measurement

and feedback. At the lowest level we have bare-bones quantum electrodynamics, which describes the interaction between atoms (qubits) and an optical probe field (ancilla) in terms of Hamiltonians.

Our publications in this area have established deep and rigorous connections among these layers, within the context of a model problem in feedback-stabilized preparation of entangled states of atomic hyperfine spins via continuous Faraday measurement and feedback via global magnetic fields. The setup is based on our group's ongoing experimental work [2] using laser-cooled Cesium atoms. In [2] and [3] we have considered the connection between the Hamiltonian-level description of such a setup and the corresponding quantum trajectory model in great detail (further work has been done very recently in [4]). In [5] and [7] we have made rigorous connections between quantum trajectory models and nonlinear stochastic control theory, and have utilized these connections to prove the absolute stability of our deterministic entanglement-generation protocols. As a result of all this work, we now have (and have disseminated via publication) a very thorough and complete understanding of feedback-stabilized entangled-state preparation via continuous measurement and real-time feedback, in an experimentally realistic model setting. This body of work provides a clear paradigm and example that can be followed in similar analyses of other experimental scenarios. In addition to providing quantum information scientists with an introduction to relevant methods from control theory, we have already seen that our work has attracted the attention of control engineers and has interested them in working on problems from quantum computation.

Quantum error correction: One of the interesting early products of our work on the state preparation problem was the realization that many features of the quantum model could in fact be captured by the classical Kalman filtering equations. The correspondence is sufficiently good that simple (linear quadratic Gaussian control-theoretic) feedback protocols based on the Kalman filter perform very well in numerical simulations based on the full quantum model (in appropriate parameter regimes). The main result of our initial work on quantum error correction with continuous syndrome measurement has been the realization that a similar situation occurs - we have seen

that the classical Wonham filtering equations play a central role in the fully-quantum problem.

Our initial results have been reported in preliminary form in [6]. In that work we describe how one can reconcile the requirements of a quantum memory application with the familiar tools of classical control theory; the main problem to be anticipated is that in a quantum memory one cannot know the actual state that is stored, whereas straightforward application of methods from classical control theory would appear to require that one know the point in phase space that one is trying to stabilize. In fact we show that one can cast the memory problem as one of stabilizing the *error* state rather than the encoded qubit state. When one shifts perspective like this, within the setting of stabilizer coding and continuous syndrome measurement, the filtering equations that pop out have precisely the classical Wonham form. This connection opens up direct possibilities for applying known techniques from hybrid control and mathematical finance to derive optimal stopping/control policies for a quantum memory, for given coding and syndrome measurement models.

[1] C. Ahn, A. C. Doherty and A. J. Landahl, Phys. Rev. A **65**, 042301 (2002).

[2] JM Geremia, J. K. Stockton and H. Mabuchi, Phys. Rev. A **73**, 042112 (2006).

[3] R. van Handel, J. K. Stockton and H. Mabuchi, J. Opt. B: Quantum Semiclass. Opt. **7** S179 (2005).

[4] L. Bouten, G. Sarma, J. K. Stockton and H. Mabuchi, to appear in Phys. Rev. A (2007); available in preprint form as quant-ph/0701224.

[5] R. van Handel, J. K. Stockton and H. Mabuchi, IEEE Trans. Automat. Control **50**, 768 (2005).

[6] R. van Handel and H. Mabuchi, quant-ph/0511221.

[7] M. Mirrahimi and R. van Handel, SIAM J. Control Optim., in press (2007); available in preprint form as math-ph/0510066.